

Nonlocal Non-Markovian Effects

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We study the nonlocal non-Markovian effects through local interactions of two subsystems and corresponding two environments. It has been found that the initial correlations between two environments, counterintuitively, the global non-Markovian effects and the local decoherence occur at the same time. We further research the nonlocal non-Markovian effects from two situations: without and with extra control. Without extra control, the abnormal phenomenon only appears under the condition that two local dynamics are non-Markovian-non-Markovian, or Markovian-non-Markovian, never appear under the condition of Markovian-Markovian. With an extra control, the abnormal phenomenon can appear under the condition of Markovian-Markovian. It shows that the correlations of two environments can make the arrival time of nonlocal non-Markovian effects become finitely shorter than the local. Next, due to observing that the classical correlations between two environments have the same function as the quantum correlations, we advise two special ways to distribute classical correlations between two environments without initial correlations. Finally, from numerical solutions in the spin star configuration, we obtain that the self-correlation of each environment helps the nonlocal non-Markovian effects.

PACS numbers: 03.67.-a, 03.65.Yz, 42.50.-p, 03.65.Ta

I. INTRODUCTION

A realistic physical system inevitably interacts with the surrounding environment, leading to lose information to the environment. If the environment can feed back the information to the system in the finite time, signifying the non-Markovian effects appear due to the environmental memory. And if the environment only feeds back the information in the infinite time, meaning the dynamics is Markovian. The dynamical process of Markovianity can also be treated as a limiting approximation of non-Markovianity[1].

It is highly interesting to explore the non-Markovian effects, because there are many systems suffering from the strong back-action from environment[2]. Hence, the non-Markovianity plays an important role in many respects. Up to now, there are a lot of work about it: non-Markovianity can assist the formation of steady state entanglement[3]; non-Markovian coherent feedback control can also suppress the decoherence[4]; non-Markovian effects are considered in a new theory of polymer reaction kinetics so that the dynamics of polymers can be controlled[5]. The direct observation of non-Markovian radiation dynamics has been completed in 3 dimension bulk photonic crystals[6]; and the observation of non-Markovian dynamics of a single quantum dot has also been completed in a micropillar cavity[7].

It is worth to note that the authors, in the Ref.[8], find a new source for quantum memory effects by the nonlocal non-Markovianity, where they utilize the quantum correlations between two environments and control the local interaction time to turn a Markovian to a non-Markovian

regime.

In this article, we further discuss about the non-local non-Markovian effects when the local dynamics are non-Markovian-non-Markovian, Markovian-non-Markovian or Markovian-Markovian. Then, we find that without extra control, the nonlocal non-Markovian effects can't appear under the condition of Markovian-Markovian. Besides the control on the interaction time in the Ref.[8], we find that reducing the strength of interaction also turn a Markovian to a non-Markovian regime. In surprise, increasing the strength of interaction can also do it. Both of two examples in the Ref.[8] they consider that the initial correlations of two environments are nonlocal(quantum correlations), because the initial state of two environments is inseparable. We find that the classical correlations can perform as well as the quantum correlations. In many real situations, the classical correlations between two environments are easier to be formed corresponding to the quantum correlations. And we advise two special ways to form the classical correlations, leading to the nonlocal non-Markovian effects at last. Finally, we investigate the non-Markovian effects in the spin star configuration under two different situations: with and without the self-correlation of each environment. We get the numerical solution, and obtain that the self-correlation of each environment helps the nonlocal non-Markovian effects.

The rest of the paper is organized as follows. In the Sec.II, we explore the reason and condition of generating the nonlocal non-Markovian effects when the local decoherence is happening. In Sec.III, We advise that use a Bell state to generate the classical correlations between two environments without the initial correlations, then, creating the nonlocal non-Markovian effects; exchanging the location of two subsystems in a proper time can also lead to the non-Markovian effects. Then, the numerical results show that the self-correlation of each environment

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helps the nonlocal non-Markovian effects in Sec.IV. Finally, in Sec.V we summarize our results and draw some conclusions.

II. MECHANISM

We consider that the initial state of two subsystems is a pure state given by

$$|\Psi_S^{12}(0)\rangle = a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle. \quad (1)$$

Under the local interactions with environments 1 and 2, a dephasing map for two qubits of of general form[8]

$$\rho_S^{12}(t) = \begin{pmatrix} |a|^2 & ab^*\kappa_2(t) & ac^*\kappa_1(t) & ad^*\kappa_{12}(t) \\ ba^*\kappa_2^*(t) & |b|^2 & bc^*\Lambda_{12}(t) & bd^*\kappa_1(t) \\ ca^*\kappa_1^*(t) & cb^*\Lambda_{12}^*(t) & |c|^2 & cd^*\kappa_2(t) \\ da^*\kappa_{12}^*(t) & db^*\kappa_1^*(t) & dc^*\kappa_2^*(t) & |d|^2 \end{pmatrix}.$$

The states of local systems $\rho_S^1(t)$ and $\rho_S^2(t)$ are fully determined by the $\kappa_1(t)$ and $\kappa_2(t)$. So if $|\kappa_1(t)|$ and $|\kappa_2(t)|$ decrease, meanwhile, $|\kappa_{12}(t)|$ or $|\Lambda_{12}^*(t)|$ increase, it is possible that the local systems lose information by decoherence, but at the same time the global system increases the information by nonlocal non-Markovian effects. Here the information of system can be quantized by the Von Neumann entropy[9]. It is obviously that no matter how fast $|\kappa_1(t)|$ and $|\kappa_2(t)|$ decrease, the nonlocal non-Markovian effects must appear in some initial states of systems when either $|\Lambda_{12}(t)|$ or $|\kappa_{12}(t)|$ increases. Because the information of the global system is also determined by $|ab^*|$, $|ac^*|$, $|ad^*|$, $|bc^*|$, $|bd^*|$, and $|cd^*|$.

We consider that the environments have continuous energy levels w and the corresponding eigenstate is $|w\rangle$ ($\hbar = 1$ throughout this article). The initial state of local system is given by

$$\begin{aligned} \rho_E^i(0) = & Z_0^i \left\{ \int_0^\infty dw \exp[-(w - w_0^i)^2] |w + c_i\rangle\langle w + c_i| \right. \\ & \left. + \int_{-\infty}^0 dw \exp[-(w + w_0^i)^2] |w + c_i\rangle\langle w + c_i| \right\}, \end{aligned} \quad (2)$$

where $w_0^i \geq 0$; $i = 1, 2$; c is a real constant and Z_0^i is the normalization coefficients.

Firstly, we discuss about the local dynamics are non-Markovian-non-Markovian. The initial correlations of two environments are the classical correlations. So, without loss of generality, we let the initial state of two environments to be

$$\begin{aligned} \rho_E^{12}(0) = & Z_1 \left\{ \int_0^\infty dw \exp[-(w - 1)^2] |w\rangle\langle w| \otimes |w\rangle\langle w| \right. \\ & \left. + \int_{-\infty}^0 dw \exp[-(w + 1)^2] |w\rangle\langle w| \otimes |w\rangle\langle w| \right\}. \end{aligned} \quad (3)$$

The interaction Hamiltonian is given by

$$H_{int}^i = g_i \int_{-\infty}^\infty dw \sigma_z^i \sigma_w^i, \quad (4)$$

where $i = 1, 2$; $\sigma_w = w|w\rangle\langle w|$; g_i is a coupling constant; and σ_z^i is the Pauli operator of system. Then,

$$\begin{aligned} |\kappa_1(t)| &= |\kappa_2(t)| = |2Z_1 \int_0^\infty dw \exp[-(w - 1)^2] \cos(2gwt)|, \\ |\Lambda_{12}(t)| &= |2Z_1 \int_0^\infty dw \exp[-(w - 1)^2] \cos(4gwt)|, \\ |\kappa_{12}(t)| &= 1, \end{aligned} \quad (5)$$

where $g = g_i$ for $i = 1, 2$ like g in the following Eq.(7) and Eq.(9). As Fig.1 shown, at time $t = 0.36$, $|\Lambda_{12}|$ begins to

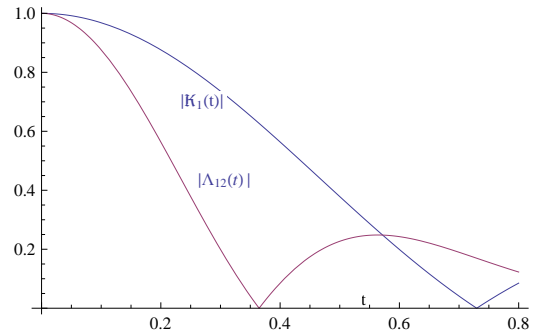


FIG. 1: From the Eq.(5), κ_1 changes with time t , in comparison with Λ_{12} . Here, the coupling constant $g = 1$.

increase when κ_1 keeps on decreasing. It means that the nonlocal non-Markovian effects appears before the local.

Secondly, we consider the local dynamics are Markovian-non-Markovian. The initial state of two environments is given by

$$\begin{aligned} \rho_E^{12}(0) = & Z_1 \left\{ \int_0^\infty dw \exp[-w^2] |w\rangle\langle w| \otimes |w + 1\rangle\langle w + 1| \right. \\ & \left. + \int_{-\infty}^0 dw \exp[-w^2] |w\rangle\langle w| \otimes |w + 1\rangle\langle w + 1| \right\}. \end{aligned} \quad (6)$$

The interaction Hamiltonian is the same as Eq.[4]. Then we obtain

$$\begin{aligned} |\kappa_1(t)| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(2gwt)|, \\ |\kappa_2(t)| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[g(2w + 2)t]|, \\ |\Lambda_{12}(t)| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[g(4w + 2)t]|, \\ |\kappa_{12}(t)| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(2gt)|. \end{aligned} \quad (7)$$

In the Fig.2, we can also see that after a while, Λ_{12}

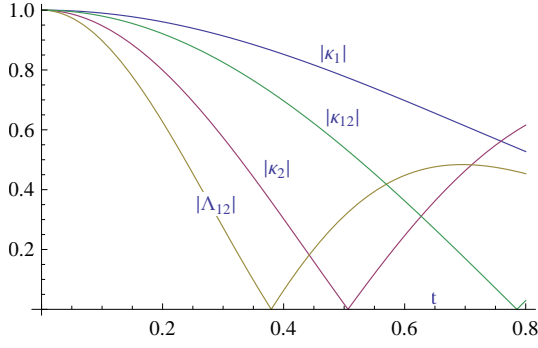


FIG. 2: From the Eq.7, $|\kappa_1|$, $|\kappa_2|$, $|\kappa_{12}|$, and $|\Lambda_{12}|$ change with time t . Here, the coupling constant $g = 1$.

begins to increase when κ_1 and κ_2 keeps on decreasing, meaning that the nonlocal non-Markovian effects occurs ahead of the local. Where, κ_1 always reduces because the dynamics of system is Markovian.

Finally, we consider the local dynamics are Markovian-Markovian. A classical initial state of two environments is described by

$$\rho_E^{12}(0) = Z_1 \left\{ \int_0^\infty dw \exp[-w^2] |w\rangle\langle w| \otimes |w\rangle\langle w| + \int_{-\infty}^0 dw \exp[-w^2] |w\rangle\langle w| \otimes |w\rangle\langle w| \right\}. \quad (8)$$

Furthermore, use the same interaction Hamiltonian (see the Eq.4) to get

$$\begin{aligned} |\kappa_1(t)| &= |\kappa_2(t)| = |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(2gwt)|, \\ |\Lambda_{12}(t)| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[4gwt]|, \\ |\kappa_{12}(t)| &= 1. \end{aligned} \quad (9)$$

From the Fig.3, we know that Λ_{12} always decrease,

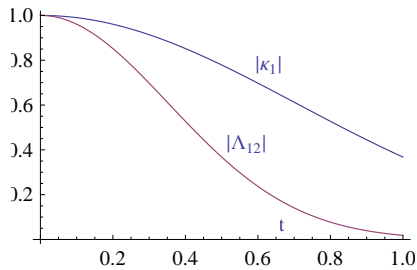


FIG. 3: From the Eq.(9), $|\kappa_1|$ and $|\Lambda_{12}|$ change with time t . Here, the coupling constant $g = 1$.

never increase. When the local dynamics are Markovian-Markovian, we can't find a initial state of two environments to make the nonlocal non-Markovian effects appear, even the existence of quantum correlations between two environments.

From the above three dynamic process and corresponding figures, we can get a conclusion that the correlations between two environments only make the arrival time of nonlocal non-Markovian effects be finitely shorter than the local. In another word, there is a upper bound on the function of the correlations between two environments. So if the local dynamics are Markovian-Markovian, the correlations can't reduce the infinite arrival time to the finite arrival time. Namely, the dynamics of global system is still Markovian when the local dynamics are Markovian-Markovian.

However, all foregoing analysis bases on without extra control (representing that the whole Hamiltonian of systems and two environments is independent of time). So if having other control, it is possible to obtain the nonlocal non-Markovian effects under the condition that the local dynamics are Markovian-Markovian.

In the Ref.[8], they control the time of two local interactions to turn a Markovian to a non-Markovian regime. Here, we find that reducing the coupling strength g_i (represent reducing the rate of local decoherence) can do it. In amazement, increasing the coupling strength can also perform as well.

Let the initial state of two environments to be the Eq.(8), the interaction Hamiltonian is given by Eq.(4). Initially, the coupling strength $g_1 = 3$ and $g_2 = 2$; when $t = 1$, reduce g_1 from 3 to 1. Then, we get

$$\begin{aligned} |\kappa_1(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(wt' + 3w)|, \\ |\kappa_2(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[2wt' + 2w]|, \\ |\Lambda_{12}(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[3wt' + 5w]|, \\ |\kappa_{12}(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(w - wt')|, \end{aligned} \quad (10)$$

where $t' = t - 1$. From Fig.4, it is obviously that $|\kappa_{12}|$ in-

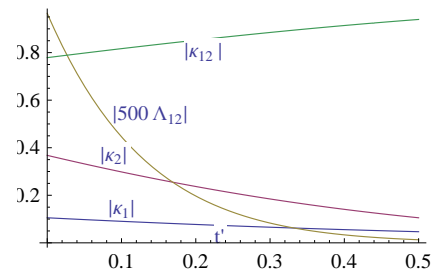


FIG. 4: From the Eq.(10), $|\kappa_1|$, $500|\Lambda_{12}|$, $|\kappa_2|$ and $|\kappa_{12}|$ change with time t' , where, in order to better compare them, we replace the $|\Lambda_{12}|$ by the $500|\Lambda_{12}|$.

creases, signifying that reducing the strength of coupling g_1 can turn the Markovian to the non-Markovian regime.

Initially, the coupling strength $g_1 = 2$ and $g_2 = 1$;

when $t = 1$, increase g_2 from 1 to 3. Then, we can obtain

$$\begin{aligned} |\kappa_1(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(wt' + 2w)|, \\ |\kappa_2(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[2wt' + w]|, \\ |\Lambda_{12}(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos[5wt' + 3w]|, \\ |\kappa_{12}(t')| &= |2Z_1 \int_0^\infty dw \exp[-w^2] \cos(w - wt')|, \end{aligned} \quad (11)$$

where $t' = t - 1$. From the Fig.5, $|\kappa_{12}|$ also increases,

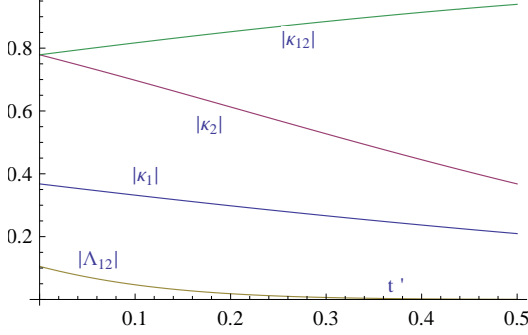


FIG. 5: From the Eq.(11), $|\kappa_1|$, $|\Lambda_{12}|$, $|\kappa_2|$ and $|\kappa_{12}|$ change with time t' .

representing that the nonlocal non-Markovian effects appears when increase the coupling strength.

III. FORMING CLASSICAL CORRELATIONS

Two nonlocal environments often haven't any correlations. It is very hard to create the quantum correlations between two macroscopical environments. But the classical correlations can be formed relatively easy. And, as the Sec.II shown, the classical correlations can perform very well for the nonlocal non-Markovian effects. Then, we advise two ways to form the classical correlations between two environments.

Firstly, we consider that a Bell state $1/\sqrt{2}(|00\rangle + |11\rangle)$ disentangles by the local interaction with two environments, where the environments are composed of bosons. Then let the system(Eq.1) locally interact with the environments.

The interaction Hamiltonian is given by

$$\begin{aligned} H_{int}^i(t) &= \chi_i(t) \sum_{k_i=1}^{n_i} g_{k_i} \sigma_z^i (b_{k_i}^\dagger + b_{k_i}) \\ &+ \chi'_i(t) \sum_{k_i=1}^{n_i} g_{k_i} \sigma_z'^i (b_{k_i}^\dagger + b_{k_i}), \end{aligned} \quad (12)$$

where the first term on the right hand side is the interaction Hamiltonian between the system and the environments; the second term is the interaction Hamiltonian

between the entanglement system and the same environments; the function $\chi'_i(t) = 1$ for $0 \leq t \leq t'$ and zero otherwise; $\chi_i(t) = 1$ for $t_i^s \leq t \leq t_i^f$ and zero otherwise; $i = 1, 2$. We denote that $t_i(t) = \int_0^t \chi_i(t') dt'$ and $t'_i(t) = \int_0^t \chi'_i(t') dt'$. The Hamiltonian of environments is described by

$$H_E^{12} = \sum_{k_1=1}^{n_1} w_{k_1} b_{k_1}^\dagger b_{k_1} + \sum_{k_2=1}^{n_2} w_{k_2} b_{k_2}^\dagger b_{k_2}. \quad (13)$$

The initial state of environments is in the thermal equilibrium state $\rho_E^{12} = 1/Z_{12} \exp(-\beta H_E^{12})$, where Z_{12} is the partition function.

Then, using weak coupling approximation $[\exp(\alpha_k b^\dagger - \alpha_k^* b), \exp(-i w_k b_k^\dagger b_k)] \approx 0$, and performing continuum limit with Ohmic spectrum density $A_i w \exp(-w/w_i)$ [10-12], we get

$$|\kappa_1| = |\exp(-\Gamma_1) \cos[\int dw A_1 \exp(-w/w_1) \xi_1]|,$$

$$|\Lambda_{12}| = |\exp(-\Gamma_1 - \Gamma_2) \cos[\sum_{i=1}^2 \int dw A_i \exp(-w/w_i) \xi_i]|,$$

in which,

$$\xi_i = 2 \sin[w_1(t_1(t) - t'_1(t))] - 2 \sin[w_1 t_1(t)] + 2 \sin[w_1 t_1(t)],$$

$$\Gamma_i = \int dw A_i \exp(-w/w_i) \coth(2w/\beta) (1 - \cos[w t_i(t)]),$$

$$\alpha_k = g_k \frac{1 - \exp(i w_k t)}{w_k}.$$

(14)

As the Fig.6 shown, the nonlocal non-Markovian appears when $|\Lambda_{12}|$ begins to increase. And we find the Bell state have the same function as the classical state $1/2|00\rangle\langle 00| + 1/2|11\rangle\langle 11|$ on forming the classical correlations of two environments, so it has the robustness against some noise. If want to form stronger correlations, it just need to increase the number of Bell state.

Then, the second way is that after two subsystems interact locally with the corresponding two environments without the initial correlations for some time, exchanging the locations of two subsystems leads to them interacting locally with each other initial environment. Similar calculation as the above way which is immune to data, it is easy to observe the nonlocal non-Markovian effects. The reason is that the system loses information, leading to the correlations between two environments. And it is necessary to exchange the locations of two subsystems because if don't exchange the locations, the correlations between the system and the two environments will keep the system losing information, never lead to the appearance of nonlocal non-Markovian effects. The benefit of this way is that it don't need other correlational states to form the correlations between two environments. Obviously, it can't form very strong correlations like the first way. Furthermore, it can't create the strong non-Markovian effects.

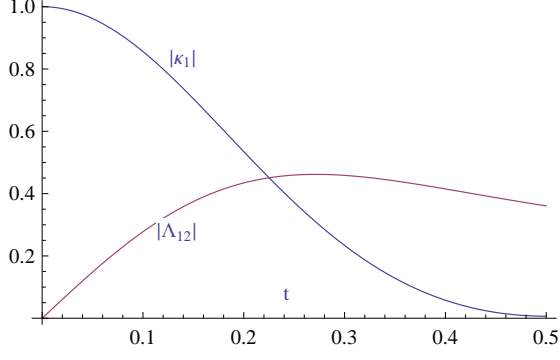


FIG. 6: From the Eq.(14), $|\kappa_1|$ and $|\Lambda_{12}|$ change with time t . Here, the parameter $\Gamma_2 = 0.5$, $A_1 = 1$, $\beta = 0.2$, $\xi_2 = \pi/2$, $t'_1(t) = t'_2(t) = 1$ and $t_1(t) = t$.

IV. SELF-CORRELATION PROMOTING THE NONLOCAL NON-MARKOVIAN EFFECTS

In this section, we explore whether the self-correlation of two environments promotes the nonlocal non-Markovian effects. We consider a simple model: the spin star configuration[10].

The interaction Hamiltonian is described by

$$H_{int} = \sum_{i=1}^2 \sum_{j=1}^{n_i} \eta_i(t) g_{ij} \sigma_{ij}^z \sigma_{iS}^z, \quad (15)$$

where $\eta_i(t) = 1$ for $t_i^s \leq t \leq t_i^f$ and zero otherwise; σ_{ij}^z and σ_{iS}^z are the Pauli operators of environment i and subsystem i respectively. The initial two environments is in the thermal equilibrium state $\rho_E^{12} = 1/Z_{12} \exp[-\beta H_E^{12}]$, in which, the Hamiltonian of two environments $H_E^{12} = \sum_{i=1}^2 B_i S_i^z + \alpha S_1^z S_2^z$. The operator $S_i^z = \sum_{j=1}^{n_i} 1/2 \sigma_{ij}^z$ for having not the self-correlation of each environment; and $S_i^z = \sum_{j=1}^{n_i} (1/2 \sigma_{ij}^z + J_i/B_i \sum_{<mn>} \sigma_{im}^z \sigma_{in}^z)$ for having the self-correlation of each environment.

Then we obtain

$$|\kappa_1(t)| = |Tr[\exp[-2i \sum_{j=1}^{n_1} \int_0^t dt' \eta_1(t') g_{1j} \sigma_{1j}^z] \rho_E^{12}]|, \\ |\Lambda_{12}(t)| = |Tr[\exp[-2i \sum_{i=1}^2 \sum_{j=1}^{n_i} \int_0^t dt' \eta_i(t') g_{ij} \sigma_{ij}^z] \rho_E^{12}]|. \quad (16)$$

Comparing the Fig.7 and the Fig.8 which are the numerical graphs of Eq.16, it is obviously that the nonlocal non-Markovian effects with the help of self-correlation is more strong, according to the growth of $\Lambda_{12}(t)$.

V. CONCLUSIONS

In this article we explore that a quantum system composed of two subsystems locally interacts with two environments in detail. We obtain that the correlations of

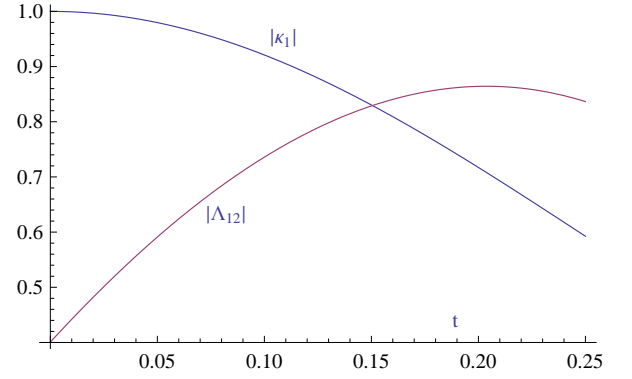


FIG. 7: $|\kappa_1|$ and $|\Lambda_{12}|$ change with time t for having the self-correlation. Here, the parameter: $n_i = 5$, $\alpha = 4$, $\beta = 0.01$, $B_i = 2$, $J_i = 10$, $\int_0^t dt' \eta_2(t') = 0.2$, $\int_0^t dt' \eta_1(t') = t$ and $g_{ij} = 1$ for $i = 1, 2$; $j = 1, \dots, 5$.

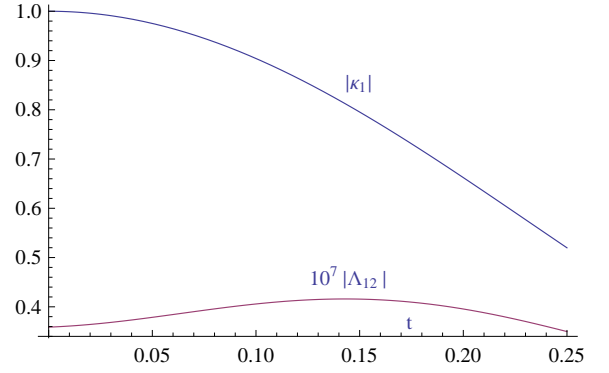


FIG. 8: $|\kappa_1|$ and $10^7 |\Lambda_{12}|$ change with time t without the self-correlation. Here, the parameter: $n_i = 5$, $\alpha = 4$, $\beta = 0.01$, $B_i = 2$, $\int_0^t dt' \eta_2(t') = 0.785$, $\int_0^t dt' \eta_1(t') = t$ and $g_{ij} = 1$ for $i = 1, 2$; $j = 1, \dots, 5$.

two environments only make the nonlocal non-Markovian effects appear in a finite time before the local when the whole Hamiltonian of system and two environments is dependent of time(without extra control), meaning that nonlocal non-Markovian effects can't appear when both of two local dynamics are Markovian. Of course, extra control can turn a Markovian to a non-Markovian regime. Besides the control of local interaction time, reducing the strength of local interaction(representing that reduce the rate of losing information locally) can make the nonlocal non-Markovian effects appear. Surprisingly, enhancing the the strength of local interaction(representing that increase the rate of losing information locally) can also make it. And we advise two ways which is easy in the experiment to form the classical correlations between two environments without initial correlations. Finally, we obtain that the self-correlation of two environments can promote the nonlocal non-Markovian effects.

Recently, the Ref.[14] accessed the non-Markovianity based on ideas of divisibilities of channels; the Ref.[15] proposed a way to quantify the memory effects in the

spin-boson model; the Ref.[16] proposed two different approaches to quantify non-Markovianity; the Ref.[17] quantified non-Markovianity via correlations. And it is also very interesting to quantify the nonlocal non-Markovianity, for a better understanding and characterization of nonlocal non-Markovian effects in more complex systems. The nonlocal non-Markovian effects mean the nonlocal memory of global environments, which may be used to perform some quantum information process, such as quantum memory[18] and quantum error

correction[19]. And this article will be meaningful for how to utilize and control the non-Markovian effects in the future works.

VI. ACKNOWLEDGMENTS

This work was supported by the National Natural Science Foundation of China under Grant No. 10975125.

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